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IMPROVED DETECTION WITH THE CROSS-AMBIGUITY FUNCTION

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The cross-ambiguity function (CAF) and the Wigner distribution (WD) are mathematically related bilinear representations of signals. The former is used in sonar and radar to locate a signal with unknown time-delay and Doppler shift. The latter characterizes the time-frequency distribution of signal and noise. This paper shows that the WD is a convenient device for data-adaptive filtering leading to improved signal-to-noise ratio in CAF detections. The proposed approach is useful when the signal competes with strong, time-varying interference.

1. INTRODUCTION

The cross-ambiguity function (CAF) is used in sonar and radar ystems to locate a signal with an unknown time delay and Doppler shift. In an active surveillance system where a signal, s(t), is transmitted and returns, with additive noise, as r(t), the time delay (T) corresponds to range and the Doppler shift corresponds to line-of-sight target velocity. Provided that the Doppler effect can be approximated by a frequency shift (ϕ), the presence of a target is indicated by a prominent peak in the CAF

$$A_{rs}(\tau,\phi) = \int s(t-\tau/2)\, r^*(t+\tau/2)\, e^{-j\phi t}\, dt \quad (1)$$

The frequency shift approximation is valid when $|(2v/c)TW| \ll 1$, where v is the target velocity, c is the propagation velocity, and TW is the time-bandwidth product of the signal. The limitation imposed by this condition is not overly restrictive. If necessary, it can be overcome by a combination of the following: limiting the signal bandwidth, limiting the integration time, or resampling r(t) with a time compression corresponding to a velocity close to the true value.

For the CAF to be useful for detection of targets and estimation of their range and velocity, the peak must be discriminated from background noise. When the noise is colored and time varying, a data-adaptive filter may be required to maximize the signal-to-noise, minimize false alarms, and improve the accuracy of the range-velocity determinations. The purpose of this note is to show that the Wigner distribution (WD) is a particularly attractive device for data filtering in this application.

2. RELATIONSHIP OF CAF AND WD

The WD of a signal, z(t),

$$W_z(t,\omega) = \int z(t-\tau/2)\, z^*(t+\tau/2)\, e^{-j\omega\tau}\, d\tau \quad (2)$$

is an elementary time-frequency representation of a one-dimensional signal. By way of various smoothing, the WD is reduced to all other time-frequency representations (e.g., spectrogram, Rihaczek). Other useful and noteworthy properties of the WD have been covered by Claasen and MecklenbrAucker (1980a, 1980b].

Of particular interest here are certain mathematical properties of the WD and ambiguity functions, as previously derived by Sussman [1962], Ackroyd [1970], and Claasen and Mecklenbrducker [1980b].

(1) The auto-ambiguity function (AAF) and the WD are a Fourier transform pair.

(2) The relationship between the magnitudesquared CAF and the AAFs of the input signals is given by:

$$|A_{rs}(\tau,\phi)|^2 = \iint e^{j(\mu\tau-\phi\nu)} A_{rr}(\nu,\mu) A_{ss}^*(\nu,\mu)\, d\nu\, d\mu \quad (3)$$

(3) The magnitude-squared CAF is a convolution of two WDs:

$$|A_{rs}(\tau,\phi)|^2 = \iint W_r(t,\omega) W_s(t-\tau,\omega-\phi)\, dt\, d\omega \quad (4)$$

A scheme by which the above relationships are used for the adaptive filtering is depicted in Figure 1. The WD of the input r (t) is modified in some manner that will enhance the signal-to-noise ratio and is Fourier-transformed to produce the AAF of the channel. Using Equation 3, that AAF is combined with a stored AAF of the transmitted signal to form

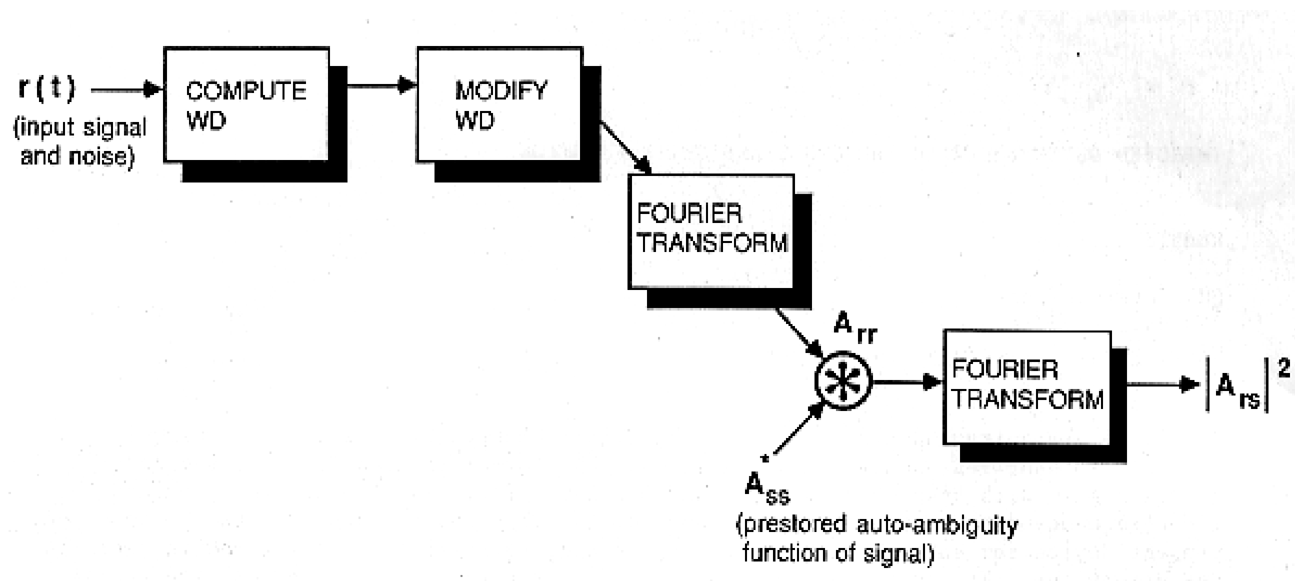

Figure I NOISE REDUCTION ALGORITHM

the magnitude-squared CAF. This procedure is efficient for large uncertainties in τ, ϕ of the target. Other variations on this algorithm may be more efficient when only a small span of τ, ϕ space is required or when one of the variables is fixed. In any case, the basic concept is to modify the WD and then employ the convolutions relationship (Equation 4) to produce the needed area of the CAF.

3. MODIFYINC THE WD

A hypothetical example of a scenario of interest is shown in Figure 2. A time-varying signal is combined with a low-level white noise and an intense, impulsive interference. In some regions of t,w space, the interference masks the signal and makes detection improbable. But since both signal and noise change with time, there are opportunities to detect the signal in intervals of time-frequency where only low noise level is encountered.

We wish to suppress the background noise in the WD to favor the signal component using the transfer function H(t, ω). The product H(t, ω)W(t, ω) is a time-varying filter on the input data. Various approaches to finding H(t,w) can be conceived. For example, Boudreaux-Bartels and Parks [1983, 1984] and Saleh and Subotic [1985] demonstrated the utility of a cookie-cutter filter, where H(t, ω) takes the value of one in the signal region and zero elsewhere. In our present problem, the location of the signal is not known a priori and may not be identifiable in a WD plot. If, however, we assume that the signal level is small relative to the interference signal, the desired transfer function is one that suppresses high-value regions the WD, passing on to the CAF calculation the low-noise areas with remaining signal information.

Our data-adaptive filter requires an estimate of noise density in every location of t,w space. The WD is not always positive, and it is not an energy distribution. A proper distribution function can be obtained by convolution of the WD with the two-dimensional Gaussian

$$G(t,\omega) = \frac{1}{\Delta t \Delta \omega} \exp\left(- \frac{t^2}{\Delta t^2} - \frac{\omega^2}{\Delta \omega^2} \right) \quad (5)$$

with the choice of At and Aw made to satisfy the equality AtAw = 1 [Janssen and Claasen, 19851. Since this convolution is a window on the AAF of the signal, one might consider an appropriate choice to be as one that minimizes the attenuation of the signal's AAF. As a bonus, this convolution is also likely to minimize WD cross-terms (Flandrin, 1984) and render a useful plot of the time-frequency energy distribution.

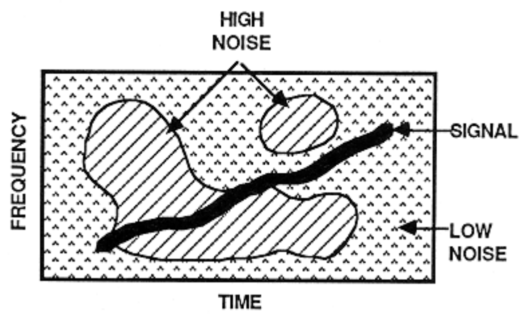

Figure 2 TIME-FREQUENCY BEHAVIOR OF SIGNAL AND NOISE

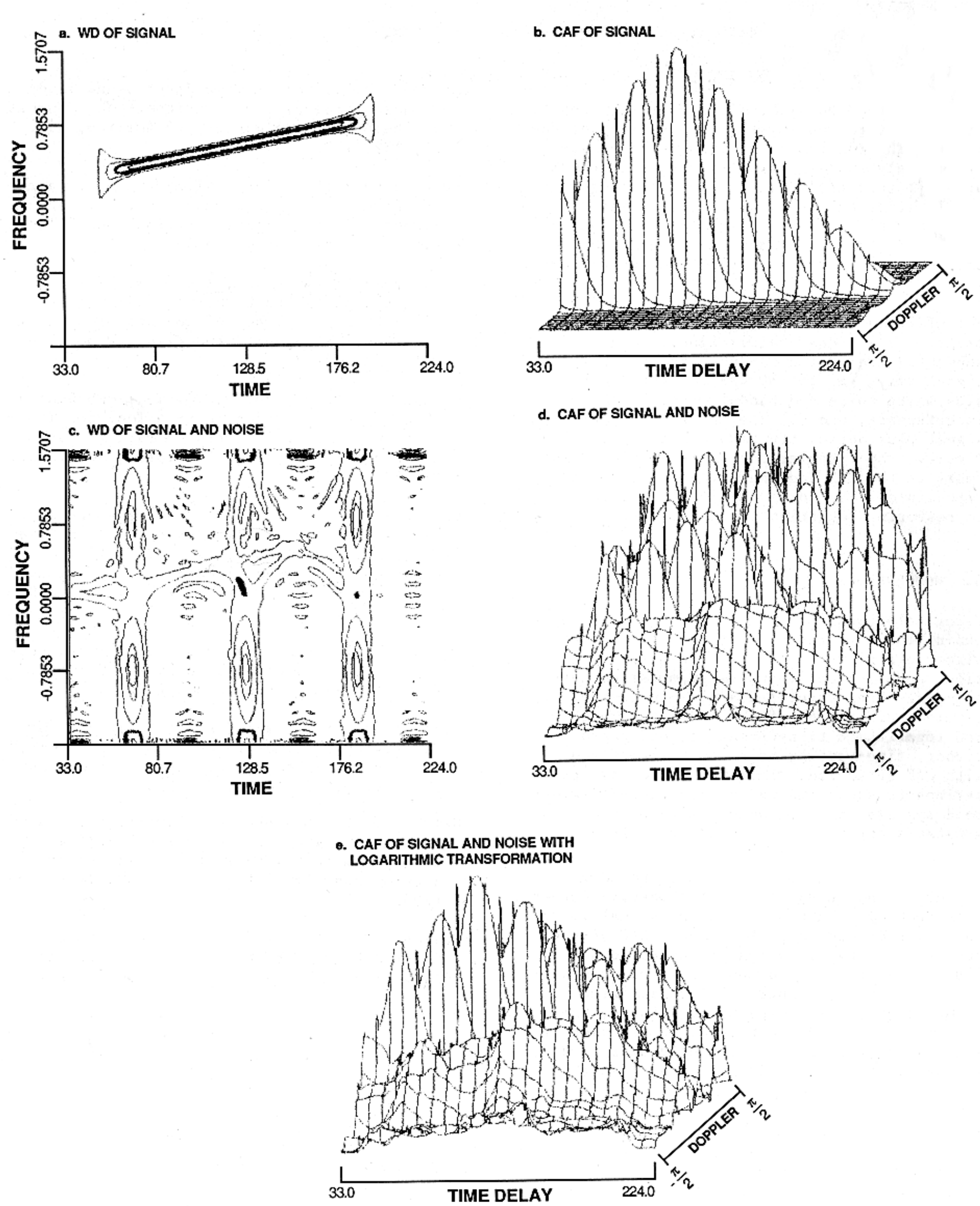

Figure 3 DEMONSTRATION

Our procedure for WD modification is given by

$$W(t,\omega) \leftarrow \log\left[1 + \frac{W(t,\omega) * G(t,\omega)}{\langle W(t,\omega) \rangle}\right] \quad (6)$$

The normalization with <W(t,w)> is required to make the operation invariant to scaling of the input. The log function output will be approximately linear for small values in the WD and will suppress large values. Although the result after the Caussian smoothing and log transformation is no longer a true WD, we are satisfied, for the reasons enumerated above, that signal structure in low-level background will be preserved.

The efficacy of this approach is shown by an example of a linear chirp signal. Figure 3 shows (1) the WD and CAF when the input is signal only, (2) the WD and CAF for signal plus white noise and band-limited impulsive interference, and (3) the CAF for the same signal plus noise, after WD noise suppression. Clearly, the interference produced high noise peaks in the CAF and made signal detection impossible. The noise suppression succeeded in restoring detection of the signal.

4. CONCLUSIONS

Using relationships inherent among the mixed two-dimensional representations, i.e., the Wigner distribution and the ambiguity function, a data-adaptive noise suppression technique has been developed and demonstrated. A simple modification of the WD, with smoothing and logarithmic transformation, mitigated strong, time-varying noise that interfered with CAF detection. This technique has applications to sonar and radar, particularly wideband systems where encounter with colored and nonstationary interference is probable.

With some obvious modifications, the same principle may be applied to the passive detection problem. In that case, the signal is not known and detection is based on a CAF of two received inputs, each containing noise and (possibly) signal. For uncorrelated noise, the WD of the two inputs will be modified independently. If noise is correlated, the time frequency modification might be based on

ï cross-WD (Ciaasen and Mecklenbrauker, 19801,

ï subject to be pursued in future research.

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